

Unit 5 Prob/Stat

Name: Answer Key

Review Normal Distribution and Percentiles (expect some z-score questions on the test as well)

Percentile Questions:

1. In a recent race Lightning McQueen placed 2<sup>nd</sup> out of 45 racers. What was his percentile compared to the rest of the racers?

How many placed below?  $45 - 2 = 43$

$$P = \frac{43}{45} * 100 = \boxed{95.6^{\text{th}} \text{ percentile}}$$

2. There are 14,000 students competing in the National Spelling Bee. You placed in the 53<sup>rd</sup> percentile. How many students placed lower than you?

$$\frac{53\%}{100} = \frac{x}{14000} * \frac{100}{100}$$

$$14000(.53) = \left(\frac{x}{14000}\right) 14000$$

$$\boxed{x = 7,420 \text{ students}}$$

3. Below is a stem-and-leaf plot for a recent test for two classes. Jenny scored an 87. Answer the following questions:

Chapter 6 Test Scores

Class A		Class B	
Stem	Leaves	Stem	Leaves
4	9	4	
5	5, 7	5	2, 7
6	6, 6, 8	6	2, 5, 8, 8
7	2, 8, 8, 8	7	2, 5
8	4, 5, 7, 8, 8	8	1, 4, 5, 7, 7
9	1, 5, 5	9	0, 1, 1, 5, 5, 5
10	0, 0	10	0

a) If Jenny is in class A, then what is her percentile when only looking at class A?

$$p = \frac{12}{20} * 100 = 60^{\text{th}} \text{ percentile}$$

b) If Jenny is in class B, then what is her percentile when only looking at class B?

$$p = \frac{11 + (2/2)}{20} * 100 = 60^{\text{th}} \text{ percentile}$$

c) If both class scores are combined, then what is Jenny's percentile?

$$p = \frac{23 + (3/2)}{40} * 100 = 61.25^{\text{th}} \text{ percentile}$$

**Normal Distribution and Table A Questions (z-scores as well) – NO EMPIRICAL RULE!!!**

For each question, construct a normal distribution curve and label the horizontal axis. Then answer each question.

4. A line for tickets to a local concert had an average (mean) waiting time of 20 minutes with a standard deviation of 4 minutes. If 2000 ticket buyers were in line, how many of them would expect to wait for more than 30 minutes?

$$\begin{aligned}\mu &= 20 \\ \sigma &= 4 \\ \text{Total: } &2000\end{aligned}$$

$$\begin{aligned}x &= 30 \\ \star(\text{more than})\star\end{aligned}$$

① Calculate the z-score:

$$z = \frac{30 - 20}{4} = 2.50 \rightarrow 2 \text{ decimal values!!}$$

② Find  $z = 2.50$  on Table A.

$$1 - .9938 = .0062$$

③ How many?

$$.0062 * 2000 =$$

About 13  
ticket buyers

5. On a recent math test, the mean score was 75 and the standard deviation was 5. Mike scored a 93.

a) What is the 25<sup>th</sup> percentile for the math scores (remember, this is the same as  $Q_1$ )?

.25 ← ① Find .25 on Table A to get z-score:

$$.2514 \rightarrow z = -0.67$$

$$\textcircled{2} \text{ Solve for } x: -0.67 = \frac{x - 75}{5}$$

$$x = 71.65$$

b) What is the 75<sup>th</sup> percentile for the math scores (remember, this is the same as  $Q_3$ )?

.75 ← ① Find .75 on Table A to get z-score:

$$.7517 \rightarrow z = 0.68$$

$$\textcircled{2} \text{ Solve for } x: 0.68 = \frac{x - 75}{5}$$

$$x = 78.4$$

c) Using the values you found in parts (a) and (b) determine if Mike's score is an outlier. Use the 1.5IQR rule

$$Q_3 = 78.4 \left\{ \begin{array}{l} IQR = Q_3 - Q_1 = 78.4 - 71.65 = 6.75 \\ \textcircled{1} Q_1 - (IQR * 1.5) = 71.65 - (6.75 * 1.5) \leq 61.525 \\ \textcircled{2} Q_3 + (IQR * 1.5) = 78.4 + (6.75 * 1.5) \geq 88.525 \end{array} \right.$$

Mike scored 93 which is greater than  $\rightarrow$  this is an outlier!

6. The Floppy Disk Company makes 3.5 inch floppy disks for disk drives that are 3.7 inches wide. The width of a manufactures disk is normally distributed with a standard deviation of 0.1 inches. The company manufactures 1000 disks every hour.

a) 30% of the disks' widths were less than how many inches?

① Find .30 on Table A:  $z = -0.52$

$x = 3.448 \text{ in.}$

② Solve for  $x$ :  $-0.52 = \frac{x - 3.5}{0.1}$

b) 45% of the disks' widths were wider than how many inches?

$1 - .45 = .55 \rightarrow$  ①  $z = -0.12$

$x = 3.488 \text{ in.}$

②  $-0.12 = \frac{x - 3.5}{0.1}$

c) In one hour, how many disks would you expect to be between 3.45 inches and 3.55 inches?

$z = \frac{3.45 - 3.5}{0.1} = -0.50$   
 $.3085$

$z = \frac{3.55 - 3.5}{0.1} = 0.50$   
 $.6915$

$.6915 - .3085 = .383$   
 $.383 * 1000 = 383 \text{ disks}$

d) The middle 40% of floppy disks are between which two widths?

$\frac{1 - .40}{2} = \frac{.60}{2} = .30$   
 $.30 + .40 = .70$

① Find .70 on Table A:  
 $z = +0.53$

② Solve both!!

$0.53 = \frac{x - 3.5}{0.1}$   
 $x_1 = 3.553 \text{ in.}$

AND  
 AND

$-0.53 = \frac{x - 3.5}{0.1}$   
 $x_2 = 3.447 \text{ in.}$

7. In an Oreo factory, the mean mass of a cookie is given as 40 g. For quality control, the standard deviation is 2 g. Cookies are rejected if they weigh more than 44 g or less than 36 g. How many cookies would you expect to be rejected in a sample of 10,000 cookies?

$$\begin{aligned} \mu &= 40 \\ \sigma &= 2 \\ x_1 &= 44 \text{ (more than)} \\ x_2 &= 36 \text{ (less than)} \end{aligned} \quad \left. \begin{array}{l} \textcircled{1} z = \frac{44-40}{2} = 2.00 \\ \textcircled{2} .0228 + .0228 \rightarrow .0456 * 10000 = \boxed{456 \text{ cookies}} \end{array} \right\}$$

$$z = \frac{36-40}{2} = -2.00$$

$$1 - .9772 = .0228$$

$$.0228$$

8. The mean life of a battery is 50 hours with a standard deviation of 6 hours. The manufacturer advertises that they will replace all batteries that last less than 38 hours. If 50,000 batteries were produced, how many would they expect to replace?

$$\begin{aligned} \mu &= 50 \\ \sigma &= 6 \\ x &= 38 \\ \text{Total} &= 50000 \end{aligned} \quad \left. \begin{array}{l} \textcircled{1} z = \frac{38-50}{6} = -2.00 \\ \textcircled{2} .0228 * 50000 = \boxed{1,140 \text{ batteries}} \end{array} \right\}$$

9. The monthly income of 5,000 workers at the Microsoft plant are distributed normally. Suppose the mean monthly income is \$1,250 and the standard deviation is \$250.

a) 20% of workers earn more than how much per month?

$$1 - .20 = .80 \quad \textcircled{1} z = 0.85$$

$$\textcircled{2} 0.85 = \frac{x - 1250}{250}$$

$$x = \$1,462.50$$

b) How many workers earn less than \$850 per month?

$$\textcircled{1} z = \frac{850 - 1250}{250} = -1.60$$

$$\textcircled{2} .0584 * 5000 = 292 \text{ workers}$$

c) What percentage of the workers earn between \$850 and \$1200 per month?

$$\textcircled{1} z = \frac{850 - 1250}{250} = -1.60$$

$$\textcircled{2} .4207 - .0584 = .3623 * 100 = 36.23\%$$

$$z = \frac{1200 - 1250}{250} = -0.20$$

d) 95% of workers earn less than how much per month?

$$\textcircled{1} .95 \rightarrow z = 1.65$$

$$\textcircled{2} 1.65 = \frac{x - 1250}{250} \rightarrow$$

$$x = \$1,662.50$$

e) The middle 50% of workers earn between how much?

$$\frac{1 - .50}{2} = .25$$

$$.25 + .50 = .75$$

$$\textcircled{1} \text{Find } .75 \rightarrow z = \pm 0.68$$

$\textcircled{2}$  solve both!!

$$0.68 = \frac{x - 1250}{250}$$

$$\text{AND} \quad -0.68 = \frac{x - 1250}{250}$$

$$x_1 = \$1,420$$

$$\text{AND} \quad x_2 = \$1,080$$

10 A bottle of fruit punch contains at least 473 ml. The machine that fills the bottles is set so that the mean volume is 477 ml. The volumes in the bottles are normally distributed.

a) What percent of the bottles are under-filled if the standard deviation is 2 ml?

$$\textcircled{1} z = \frac{473 - 477}{2} = -2.00 \quad \textcircled{2} \text{Table A: } .0228 * 100 =$$

2.28%

b) What percent of the bottles are under-filled if the standard deviation is 4 ml?

$$\textcircled{1} z = \frac{473 - 477}{4} = -1.00 \quad \textcircled{2} \text{Table A: } .1587 * 100 =$$

15.87%



11. A grading scale is set up for 1000 students' test scores. It is assumed that the scores are normally distributed with a mean score of 75 and a standard deviation of 15

a) How many students will have scores between 50 and 70?

$$\textcircled{1} z = \frac{50 - 75}{15} = -1.67 \quad \textcircled{2} .3707 - .0475 = .3232$$

$$z = \frac{70 - 75}{15} = -0.33$$

$$.3232 * 1000 = \boxed{323 \text{ students}}$$

b) If 84.1% of students pass the test, what was the minimum passing score?

$$1 - .841 = .159$$

↑ more than

① Find .159 on Table A:

$$z = -1.00$$

$$\textcircled{2} -1.00 = \frac{x - 75}{15}$$

$$\boxed{x = 60}$$