5.0 - 5.1 Notes

An Introduction to Polynomials

**Vocabulary**

**Monomial** - one term
Ex: $5x^3$

**Constant** - a number (no variable)
Ex: $3$ or $-2$

**Polynomial** - more than one term
Ex: $x^2 + 2x + 7 - 3x^4$
More Vocabulary

The monomials that make up a polynomial are called the [terms] of the polynomial.

In the polynomial $x^2 + 2x + x + 4$, the monomials 2x and x can be combined because they are [like terms]. The result is $x^2 + 3x + 4$.

The polynomial $x^2 + 3x + 4$ is a [trinomial] because it has 3 unlike terms.

A polynomial such as $5y^3 + y^2$ is a [binomial] because it has 2 unlike terms.

The degree of a polynomial is the degree of the monomial with the greatest degree. For example, the degree of $x^2 + 3x + 4$ is 2 and the degree of $5y^3 + y^2$ is 3.
Monomials

\[ x, \ 2, \ 2x^4, \ \sqrt{3}x, \ \frac{x}{3}, \ 3x^2 \]

Not Monomials

\[ \sqrt{x} \rightarrow \text{no variables under radicals} \]
\[ \frac{1}{x} \rightarrow \text{no variables in denominators} \]
\[ x^{-2} \rightarrow \text{no negative exponents} \]
Example 1: Which of the following are polynomials?
If it's a polynomial, state the degree.

a) \( x^2 - 6x + 2x^3 + 3 \)
   Yes, \( \text{deg} = 3 \)

b) \( x^6 - 4x^3 - \frac{2}{x^3} \)
   No

(c) \( (2x-8)(x-4)^3 \)
   \( (2x-8)(x-4)(x-4)(x-4) \)
   No

(d) \( 3 - \sqrt{x} \)
   No

\( 2x^4 \) + ... +

Yes, \( \text{deg} = 4 \)
Classifying Polynomials: We classify polynomials by the **number of terms** and the **degree**. Complete the chart below.

<table>
<thead>
<tr>
<th>Polynomial Example</th>
<th>Degree</th>
<th>Name using Degree</th>
<th>Number of Terms</th>
<th>Name using Number of Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>Constant</td>
<td>1</td>
<td>monomial</td>
</tr>
<tr>
<td>x + 3</td>
<td>1</td>
<td>Linear</td>
<td>2</td>
<td>binomial</td>
</tr>
<tr>
<td>3x^2</td>
<td>2</td>
<td>Quadratic</td>
<td>1</td>
<td>monomial</td>
</tr>
<tr>
<td>2x^3 − 5x^2 − 2x</td>
<td>3</td>
<td>Cubic</td>
<td>3</td>
<td>trinomial</td>
</tr>
<tr>
<td>x^4 + 3x^2</td>
<td>4</td>
<td>Quartic</td>
<td>2</td>
<td>binomial</td>
</tr>
<tr>
<td>−2x^5 + 3x^2 − x + 4</td>
<td>5</td>
<td>Quintic</td>
<td>4</td>
<td>polynomial of 4 terms</td>
</tr>
</tbody>
</table>
More Vocabulary

Standard Form - A polynomial is written in **standard form** when

- the terms are arranged by degree in descending number order
- all coefficients are real numbers
- all exponents are non-negative integers

Using the example \(7x^3 + x - 2x^5 + 3\)

In **standard form** this would be written as \(-2x^5 + 7x^3 + x + 3\)

The **leading term** is \(-2x^5\)

The **leading coefficient** is \(-2\)

The **degree** is \(5\)
Example 2: Write each polynomial in standard form and fill in the blanks below.

a. \( \frac{12x^2 + 9x}{3} = 4x^2 + 3x \)

   Standard form: \( 4x^2 + 3x \)
   
   Leading term: \( 4x^2 \)
   
   Leading coefficient: \( 4 \)
   
   Degree: \( 2 \)
   
   Classify by degree: Quadratic
   
   Classify by number of terms: Binomial

b. \( 5x^2 - x^4 + 6x \)

   Standard form: \( -x^4 + 5x^2 + 6x \)
   
   Leading term: \( -x^4 \)
   
   Leading coefficient: \( -1 \)
   
   Degree: \( 4 \)
   
   Classify by degree: Quartic
   
   Classify by number of terms: Trinomial
Operations with Polynomials

**Example 3:** Given

\[
\begin{align*}
  f(x) &= x^2 - 3x + 1 \\
  g(x) &= 4x + 5
\end{align*}
\]

Find \( f(x) + g(x) \) and \( f(x) - g(x) \)

<table>
<thead>
<tr>
<th></th>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) + g(x) )</td>
<td>((x^2 - 3x + 1) + (4x + 5))</td>
<td>((x^2 - 3x + 1) - (4x + 5))</td>
</tr>
<tr>
<td></td>
<td>( x^2 - 3x + 1 + 4x + 5 )</td>
<td>( x^2 - 3x + 1 - 4x - 5 )</td>
</tr>
<tr>
<td></td>
<td>( x^2 + x + 6 )</td>
<td>( x^2 - 7x - 4 )</td>
</tr>
</tbody>
</table>
Example 4: Given

\[
\begin{align*}
  f(x) &= x^2 + 5x - 1 \\
  g(x) &= 3x - 2
\end{align*}
\]

Find \( f(x) \cdot g(x) \) and \( f(x) \div g(x) \)

### Multiplication

\[
 f(x) \cdot g(x) = (x^2 + 5x - 1)(3x - 2)
\]

\[
 = (3x - 2)(x^2 + 5x - 1)
\]

\[
 = 3x^3 + 15x^2 - 3x - 2x^2 - 10x + 2
\]

\[
 = 3x^3 + 13x^2 - 13x + 2
\]

### Division

\[
 f(x) \div g(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 5x - 1}{3x - 2}
\]
Example 5: Given

\[ f(x) = 4x + 2 \]
\[ g(x) = 4x - 1 \]

Find the following.

**Addition**

\[ f(x) + g(x) = (4x+2) + (4x-1) = 4x+2 + 4x - 1 = 8x + 1 \]

**Subtraction**

\[ f(x) - g(x) = (4x+2) - (4x-1) = 4x+2 - 4x + 1 = 3 \]

**Multiplication**

\[ f(x) \cdot g(x) = (4x+2)(4x-1) = 16x^2 - 4x + 8x - 2 = 16x^2 + 4x - 2 \]

**Division**

\[ f(x) \div g(x) = \frac{f(x)}{g(x)} = \frac{4x+2}{4x-1} \]
Example 6: Given \( f(x) = -x^2 + 2x + 5 \)

Find \( f(2) \)

\[
\begin{align*}
= & - (2)^2 + 2(2) + 5 \\
= & -4 + 4 + 5 \\
= & 5
\end{align*}
\]

Find \( f(-2) \)

\[
\begin{align*}
= & - (-2)^2 + 2(-2) + 5 \\
= & -4 - 4 + 5 \\
= & -3
\end{align*}
\]
Homework: 5.0 - 5.1 worksheet (1 - 9, 21 - 24)

Below are two problems from the HW worksheet.

1. $5x + 2$
   - Degree: 1
   - Linear
   - 2 terms
   - Binomial

8. $\frac{2x^4 + 4x - 5}{y}$
   - $\frac{1}{2}x^4 + x - \frac{5}{4}$
   - Quartic
   - Trinomial